CMS Session on History of Mathematics
University of Waterloo

9 December 2017

Overloading and Information Hiding in 1906

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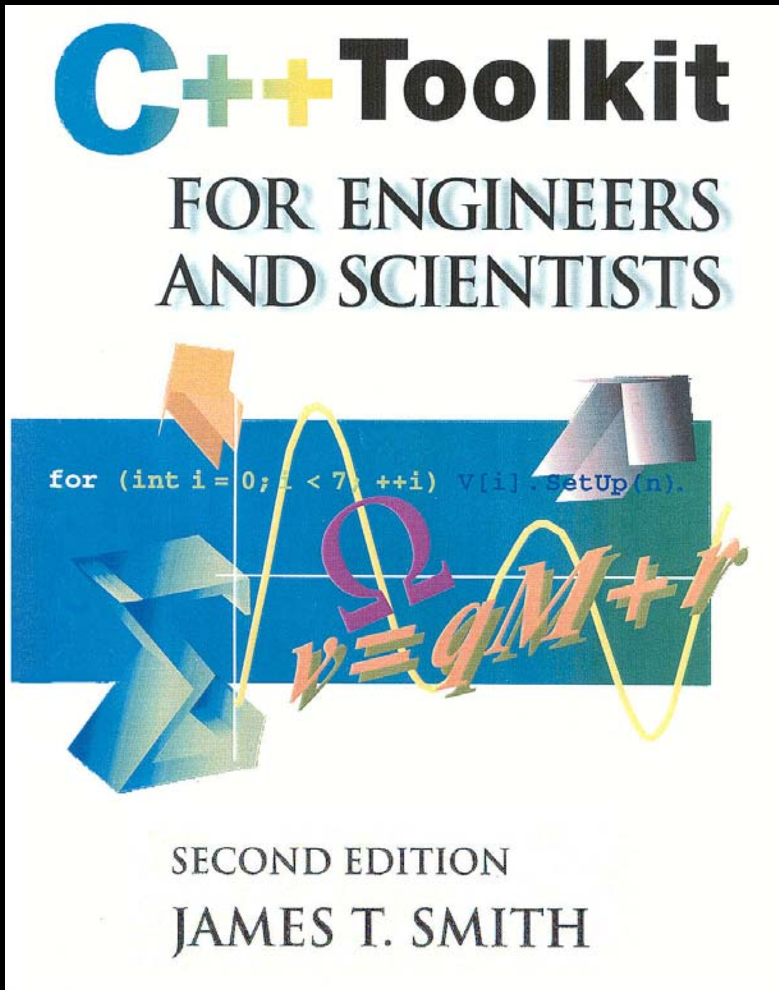
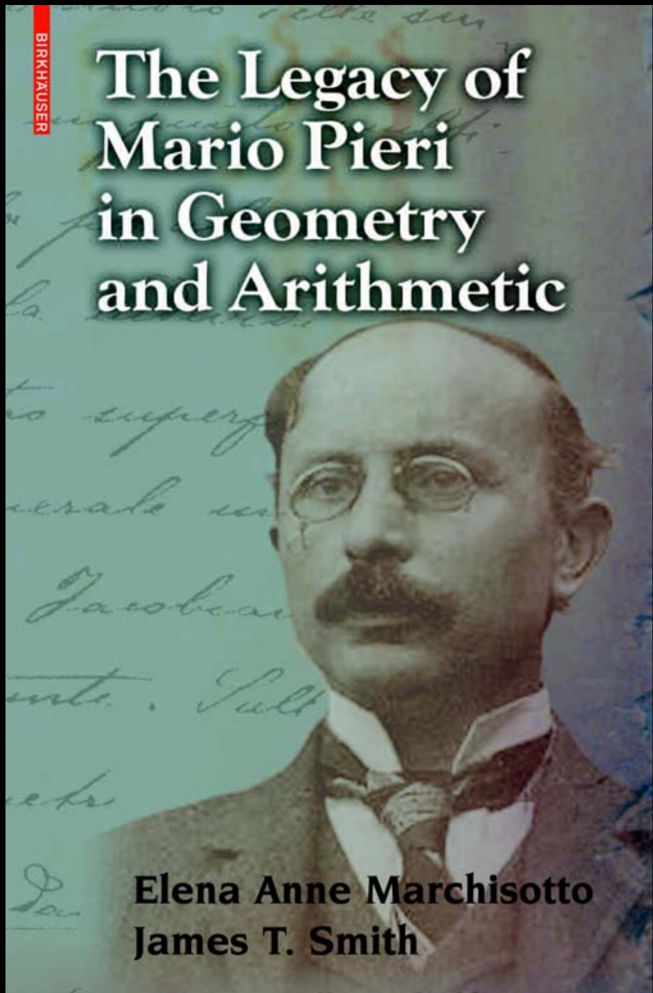
San Francisco State University



Springer, 1999

My talk *ends* on this topic.

But it *begins* with work on the next slide.



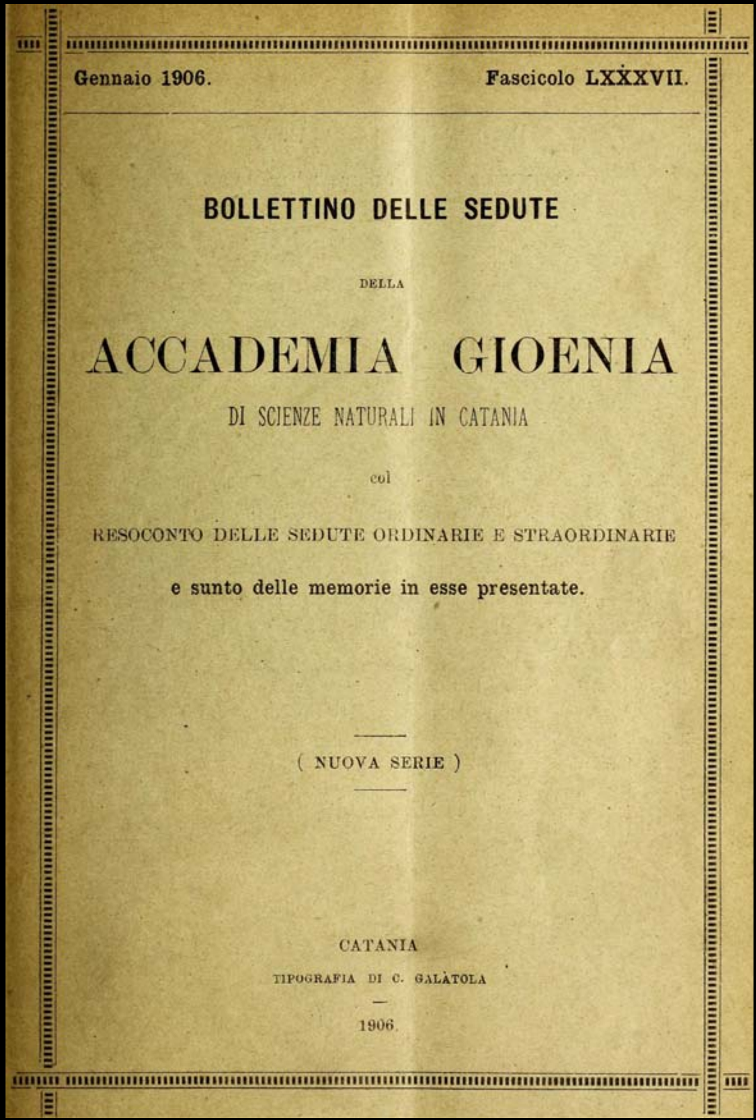
Birkhäuser, 2007

The Legacy of
Mario Pieri
in Foundations
and Philosophy
of Mathematics

Elena Anne Marchisotto
Francisco Rodríguez-Consuegra
James T. Smith

In preparation

- Mario Pieri (1860–1913)
- Algebraic & enumerative geometry
- Foundations and philosophy of mathematics
- Obscure paper anticipates developments 50+ years later.
- 1906. *On an Arithmetical Definition of the Irrationals*.



Introducing Real Numbers

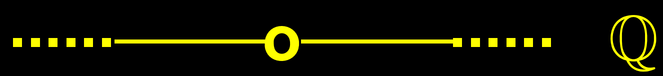
- **Weierstrass, 1878: equivalence classes of formal sums of quotients of whole numbers**

$$\sum_{k=0}^{\infty} \frac{n_k}{d_k} \qquad d_k > 0$$

- **Cantor, 1872: equivalence classes of "Cauchy" sequences $\langle x_k \rangle$ of rationals**

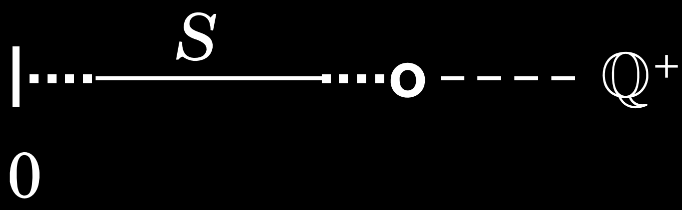
$$(\forall \varepsilon > 0)(\exists k)(\forall m, n \geq k)$$
$$|x_m - x_n| < \varepsilon$$

- **Dedekind, 1872: cuts in rationals**



Moritz Pasch, 1882:

- $\mathscr{S}$ = **family of nonempty**



$p < q \in S \;\Rightarrow\; p \in S$
$p \in S \;\Rightarrow\; (\exists q \in S)\; p < q$

- **For** $S, T \in \mathscr{S}$ **define**

$S < T \;\Leftrightarrow\; S \subsetneq T$

$S + T \;=$
$\{\, s + t \;:\; s \in S \;\&\; t \in T \,\}$

etc.

EINLEITUNG

IN DIE

DIFFERENTIAL- UND INTEGRAL-
RECHNUNG

VON

DR. MORITZ PASCH,
PROFESSOR AN DER UNIVERSITÄT ZU GIESSEN.

LEIPZIG,
VERLAG VON B. G. TEUBNER.
1882.

Pasch 1882, 11:

Rational segments

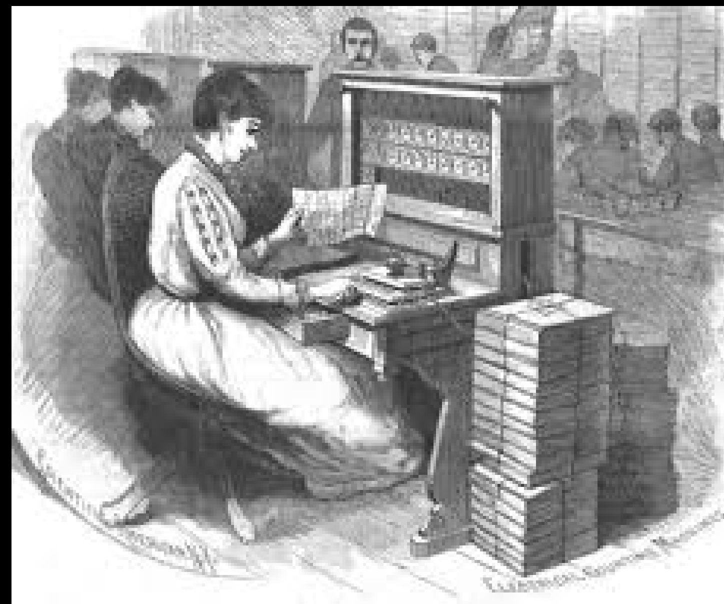
$$S_q = \{p \in \mathbb{Q}^+ : p < q\}$$

have the same properties as the rationals $q \in \mathbb{Q}^+$.

The other segments have the properties we want for the irrationals.

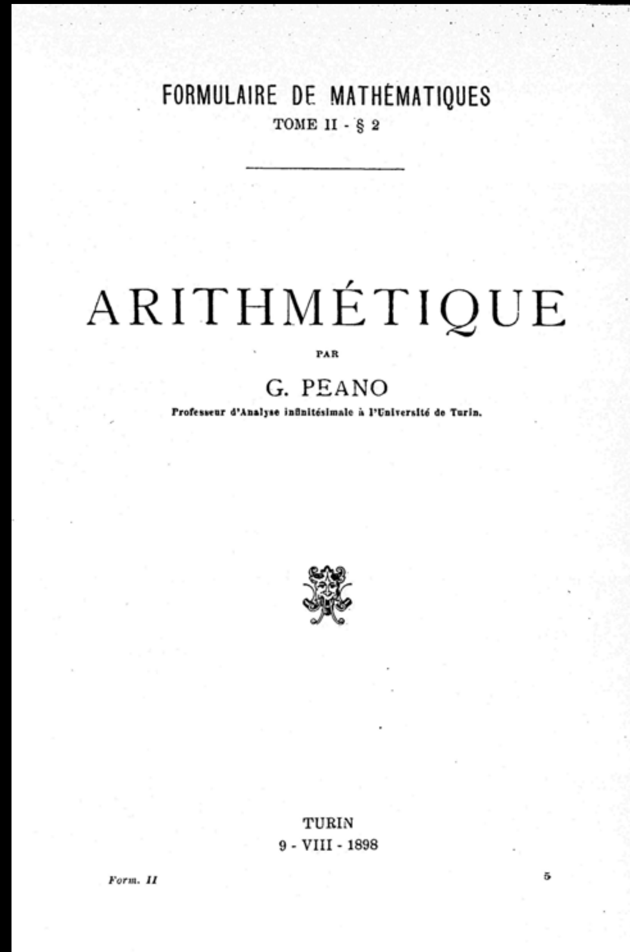
From now on *number* will mean *segment*. Some numbers are rational and the others, *irrational*.

Meanwhile...



Tabulating the 1890 U. S. Census

Giuseppe Peano
(1858–1932)



FORMULAIRE DE MATHÉMATIQUES
TOME II - § 2

ARITHMÉTIQUE

PAR

G. PEANO

Professeur d'Analyse infinitésimale à l'Université de Turin.

TURIN
9 - VIII - 1898

Form. II 5

1898

- **Peano tried to put *all* math into a *single* system.**
- **His definition of $s+t$ was**

» «

$\S 2 \quad + \; N_1$ 9

Quelques Auteurs appellent « puissance » d'une opération l'opération répétée. (P016·1) « Deux puissances d'une même opération sont commutables entre elles ».

(P017). « Des propositions 014 et suivantes, par la substitution de N_0 et $+$ à la place de s et u découlent les propositions 011 etc. sur l'addition ».

019. $s,t,u \; \varepsilon \; \mathrm{Cls}`N_0 \,.\, a \; \varepsilon \; N_0 \,.\, \supset .$

·1. $s+ = x \ni [\exists \; s \cap y \ni (x = y+)]$ Df

·2. $s+a = x \ni [\exists \; s \cap y \ni (x = y+a)]$ Df

·3. $a+s =$ » » $(x = a+y)]$ Df

·4. $s+t = x \ni [\exists (y, z) \ni (y \varepsilon s \,.\, z \varepsilon t \,.\, x = y+z)]$ Df

·5. $a+s = s+a = \iota a + s$

·6. $s+t = t+s$ ·7. $s+(t+u) = (s+t)+u = s+t+u$

·8. $N_0 + N_0 = N_0$

Ex.: N_0+ signifie « successif de quelque nombre »; cette classe est indiquée par N_1 P020. N_0+a P152·2; $a+N_0$ P046; $s+t$ P084·1.Voir F_2§1 P530 note.

$N_0 \; + \; N_1$

020. $N_1 = N_0 + \quad \{ =$ « nombre positif » $\}$ Df

·1. $N_1 \supset N_0$ [= P002·2]

·2. $0 \, \mathrm{-}\varepsilon \, N_1$ [= P002·4]

·3. $N_0 = \iota 0 \cup N_1$

[$0 \; \varepsilon \; \iota 0 \cup N_1$ (1)

$x \, \varepsilon \, N_0 \,.\supset.\, x+ \, \varepsilon \, N_1$ (2)

(1) . (2) . Induct $.\supset.\; N_0 \supset \iota 0 \cup N_1$ (3)

P002·1 . P·1 $.\supset.\; \iota 0 \cup N_1 \supset N_0$ (4)

(3) . (4) $.\supset.$ P]

·4. $N_1 = N_0 - \iota 0$ [P·3 . P·2. §1P352 $.\supset.$P]

·5. $a \, \varepsilon \, N_0 \,.\, b \, \varepsilon \, N_1 \,.\supset.\; a+b \, \varepsilon \, N_1$

- **In an 1899 paper he noted: by his rules, $S \in \mathcal{S} \Rightarrow$**

$$S^2 = \{ s^2 : s \in S \} \notin \mathcal{S},$$

- **but we want**

$$S \times S = \{ s \times t \; : \; s,t \in S \}.$$

- **We'd need a new op $S \uparrow 2$.**
- **And he didn't like that:**

Peano, Giuseppe. 1899. Sui numeri irrazionali. *Rivista di matematica* 6: 126–140.

"It is possible, speaking always of segments, to construct a complete theory of the irrationals; but the formulas, if it is desired to exclude all danger of ambiguity, would be presented in a form altogether different from that in use today in algebra. While admitting that this form is appropriate for the civilizations in which we live,…it is altogether necessary that our notations agree with those used by everyone."

Russell, Bertrand. 1903. *Principles of Mathematics*, 286.

"…there is no logical ground for distinguishing segments of rationals from real numbers. If they are to be distinguished, it must be in virtue…of some wholly new axiom…The above theory, on the contrary, requires no new axiom:…an irrational actually *is* a segment of rationals which does not have a limit."

Bertrand Russell
(1872–1970)

Pieri, Mario. 1906. Sopra una definizione aritmetica degli irrazionali. *Bollettino delle sedute della Accademia Gioenia di Scienze Naturali in Catania* 87: 14–22.

"This [fault] deserves to be acknowledged and possibly removed.…It suffices to identify each irrational (or real) number not just with such a segment…but with the same class *considered as…an individual*…. [This new] notion might be a most useful tool…."

Pieri's Suggestion

New logical operator: I

New logical axioms:

- S **is a class of individuals (non-classes)**
  $\Rightarrow$ IS **is an individual & $IS \notin S$.**
- S, T **are classes of individuals** $\Rightarrow$ $(IS = IT \Leftrightarrow S = T)$**.**

New definitions:

- $\mathbb{R}^+ = \{IS : S \in \mathcal{S}\}$
- **for** $x, y \in \mathbb{R}^+$,
  $x < y \Leftrightarrow (\exists S, T \in \mathcal{S})(x = IS \;\&\; y = IT \;\&\; S \subsetneq T)$
- **etc.**

Lack of impact

- **Pieri 1906 attracted *virtually no attention*,**
- **even though it could be extended to yield various *types* of individuals.**
- **Mathematics progressed in another way: today's familiar informal modular approach.**
- **Peano's project lapsed.**
- **……..**

Fifty years later:

UNIVAC I
Sold to the Census

Peano & Pieri

- Grand system for all math
- For use in instruction...
- Ideography was a tool.
- Familiar notation for high-level operators,
  - with arguments of varying but analogous types,
  - without colliding with lower-level usage...
- Hide lower-level detail !

Object-Oriented Programming

- Large-scale electronic manipulation of data of different but analogous types, using different algorithms for analogous operations...
- Overloading enhances reliability by making programming more intuitive.
- Information hiding fosters portability and prevents disruption of low-level computations by higher-level software errors.

- The aim of the C++ class concept ... is to provide the programmer with a tool for creating new types that can be used as conveniently as the built-in types.
- The fundamental idea in defining a new type is to separate the incidental details of the implementation ... from the properties essential to the correct use of it.
- There are several benefits to be obtained from restricting access to a data structure to an explicitly declared list of functions.

Bjarne Stroustrup
(1950–)

The C++ Programming Language. 2nd edition. Addison-Wesley, 1991. 143–146.

I hope some historians will pursue this parallelism more deeply!

Thank you for your interest!

James T. Smith
Professor Emeritus
San Francisco State University

Overloading and Information Hiding in 1906

James T. Smith

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